# LAB # 1

## ( OVERVIEW OF THE COMPUTATIONAL SYSTEM MAPLE)

1. This exercise focuses on the manipulation of prime numbers and using Maple's help system. Answer the following questions:
   1.1. Obtain a 12 digit integer generated randomly.
   1.2. Decide, without factorizing, whether $a$ is prime.
   1.3. Factorize $a$.
   1.4. Repeat exercises 3.2 and 3.3 for fifty numbers, obtained randomly.
   1.5. Obtain the primes which are anterior and posterior to the number 123456789.
   1.6. Calculate the thousand-th prime number.

2. 2.1. Compute the g.c.d. of $2^{30} - 1, 31!, 4^{14} - 1$.
   2.2. Obtain the coefficient of $x^{20}$ in the polynomial $f(x, y, z) = (5x^2y + y^2 + 12xz)^{20}$.
   2.3. Obtain all the coefficients of the above mentioned polynomial in the variables $\{x, y\}$.
   2.4. Calculate the g.c.d. of the coefficients of the polynomial $f(x) = 12x^3 + 120978x^2 + 1249875000x + 12300$.

3. 3.1. Calculate the g.c.d. over $\mathbb{Q}$ of the polynomials $f_1(x) = x^4 - 2x^2 - 9x + 10, f_2(x) = x^4 - 3x^3 - 8x^2 - 15x + 25, f_3(x) = x^4 - 2x^3 - 6x^2 - 13x + 20$.
   3.2. Factorize the previous polynomials of over $\mathbb{Q}$
   3.3. Calculate the g.c.d. over $\mathbb{Q}$ of the coefficients of the polynomial $f(x, y, z) = 79x^2yt + 56xy^2z + 49x^2y^3 + 63x^2y^2z + 57x^2z^3 - 59y^2z^3$ in the variable $z$.

4. Matrix manipulation.
   4.1. Built a $3 \times 6$ matrix $A = (a_{i,j})$ where $a_{i,j} = \frac{i}{i+j}$.
   4.2. Built a $4 \times 4$ matrix $C = (c_{i,j})$ where $c_{i,j} = max\{i, j\}$.
   4.3. Built a $4 \times 4$ matrix $D = (d_{i,j})$ where $d_{i,j}$ is the $(i + j)$-th prime number.
   4.4. Compute $C + D, C \cdot D, 5\,C + \frac{3}{2}\,D, C^{-1}, D^{-5}, C^3$ and $(x^2 + 1) \cdot C$.
   4.5. Compute the determinant of $C$ and $D$. Compute the rank of $A$ and $B$.
   4.6. Obtain the formula of the determinant of a $4 \times 4$ matrix.

5. Let us consider

$$f(x) = \frac{3x^2 - 2x + \sqrt{5}}{4x^2 + 3\sqrt{2}x - 0.66}$$

   Answer the following questions:
   5.1. Evaluate $f(x)$ for $x = 1, 10, \ldots, 10^9$.
   5.2. Represent $f$ graphically.
   5.3. Calculate the limit of $f(x)$ when $x$ tends to $\infty$.

6. Let us consider $f(x) = \frac{senx}{x}$. Answer the following questions:

6.1. Calculate $f'(x)$.

6.2. Calculate $f^{v)}(x)$:

6.3. Obtain $f^{100)}(\pi)$.

6.4. Obtain the 15 first digits of the value obtained in 6.3.

6.5. Check by integration that the result obtained in 6.2 is correct.

7. Given the plane curve, in polar coordinates, $r = 1 + acos(\theta)$, represent it graphically for $a = 1, 2, 3, 4, \frac{1}{2}$.

8. Represent the level curves of the function $f(x, y) = x^2 - x^4 - y^2$.

9. Simplify the following expressions:

9.1. $(x-y)^3 + 3(x-y)(x+y) + (x+y)^3 + 3(x-y)(x+y)^2 - 8x^3 + 3(x-y)(x-1-y)(x+y)$

9.2. $\frac{x^{100}-1}{x-1} + \sum_{i=0}^{99} x^i$

9.3. $\frac{3}{cos^2 x} - \frac{2}{cos^6 x} + \frac{3sen^2 x}{cos^6 x} - tg^6 x$

9.4. Let us consider $f = x^3y^2 + z, g = 4x^5y^2 + z^3 + 1, h = 2x^3y^4 + 7z^3$. Simplify $\frac{(f+g)^2+h^3}{f+g+h}$.