ROTOR AERODYNAMICS

Modified Momentum Theory
Vertical Descent Flight
Initial Thoughts:

In vertical descending flight:

- The flight velocity is in the upwards direction.
- The induced velocity on the disc plane acts in the same direction as in climbing flight, which is, downwards.
- The total velocity on the disc plane can have a positive or negative value.
- Applying calculations based on the modified momentum theory, the results are not acceptable.
ROTOR AERODYNAMICS

VERTICAL DESCENDING FLIGHT

MODIFIED MOMENTUM THEORY.

- Diagrams of the different regimes of flight
- Modified Momentum Theory. Induced Power.
- Vertical autorotation.
FLIGHT REGIMES

This theory presents 6 different regimes of vertical flight.

These regimes keep the hypothesis’ presented in chapter 2 (Momentum Theory in vertical climbing flight), according to the following ranges:

\[
|V_v| \leq v_i \quad ; \quad v_i \leq |V_v| \leq 2v_i \quad ; \quad |V_v| \geq 2v_i
\]
FLIGHT REGIMES

Vertical Climb

\[ V_v > 0 \]
\[ v_i > 0 \]
\[ V_v + v_i > 0 \]
\[ V_v + 2v_i > 0 \]

\[ P_i = T \left( V_v + v_i \right) > 0 \]
FLIGHT REGIMES

Hover flight

\[
V_v = 0 \\
\nu_i = \nu_{io} > 0 \\
V_v + \nu_i = \nu_{io} > 0 \\
V_v + 2 \nu_i = 2 \nu_{io} > 0 \\
P_i = P_{io} = T \nu_{io}
\]
FLIGHT REGIMES

Vortex Rings

\[ V_v < 0 \quad (|V_v| < v_i) \]

\[ v_i > 0 \]

\[ V_v + v_i > 0 \]

\[ V_v + 2v_i > 0 \]

\[ P_i = T( V_v + v_i ) > 0 \]
FLIGHT REGIMES

Autorotation

\[ V_v < 0 \ (|V_v| = v_i) \]

\[ v_i > 0 \]

\[ V_v + v_i = 0 \]

\[ V_v + 2v_i = v_i > 0 \]

\[ P_i = T (V_v + v_i) = 0 \]
FLIGHT REGIMES

Turbulent Wake

\[ V_v < 0 \ ( v_i < |V_v| < 2 \ v_i ) \]
\[ v_i > 0 \]
\[ V_v + v_i < 0 \]
\[ V_v + 2 \ v_i > 0 \]
\[ P_i = T ( V_v + v_i ) < 0 \]
FLIGHT REGIMES

Windmill brake

\[ V_v < 0 \quad (|V_v| > 2 \ v_i) \]
\[ v_i > 0 \]
\[ V_v + v_i < 0 \]
\[ V_v + 2 \ v_i < 0 \]
\[ P_i = T \left( V_v + v_i \right) < 0 \]

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MMT. DESCENDING VERTICAL FLIGHT
New concepts:

- In the first 3 regimes it is necessary to provide power to rotate the rotor.

![Diagram](image)
New concepts:

- In the fourth regime depicted, the rotor turns without absorbing power from the power supply or producing power.
New concepts:

- In the last two regimes the rotor turns without the need for a power supply.
New concepts:

- In other regimes the configuration of the streamlines are clearly absurd.
New concepts:

- The Momentum Theory is not applicable in these conditions.
INDUCED POWER CALCULATION

The aim of this section is to apply the equation of momentum to the corresponding regimes of $V_V \geq 0$ and the windmill brake, and describe some empirical expressions.

These expressions have to be adapted to the previous regimes, matching with what occurs in the regimes of the vortex rings, autorotation and windmill brake.

The following expressions are used:

\[ \frac{V_v}{v_{io}} = \bar{V}_v, \quad \frac{v_i}{v_{io}} = \bar{v}_i, \quad \frac{V_v + v_i}{v_{io}} = \bar{U}_p \]
INDUCED POWER CALCULATION

Vertical Climb, Hover.

\[ \frac{V_v + v_i}{v_{io}} \cdot \frac{v_i}{v_{io}} = 1 \]

\[ \begin{aligned} \bar{V}_V &= \frac{1}{\bar{V}_i} - \bar{v}_i \\
\bar{V}_V &= \bar{U}_P - \frac{1}{\bar{U}_P} \end{aligned} \]

\[ \bar{V}_v = \bar{U}_P - \frac{1.2}{\bar{U}_P} \]

\[ \bar{P}_i = \frac{1}{2} \bar{V}_v + \sqrt{1.2 + \frac{1}{4} \bar{V}_v^2} \]
INDUCED POWER CALCULATION

Regime of vortex rings

\[ 0 < \bar{U}_p < 1.1 \quad \text{y} \quad -1.7 < \bar{V}_v < 0 \]

\[ 0 < \bar{U}_p < 0.8 \quad - \]

\[ \bar{V}_v (\bar{V}_v + 1.18) = 0.812 + 0.072 \bar{U}_p - 1.75 \bar{U}_p^2 \]

\[ 0.8 < \bar{U}_p < 1.1 \quad _- \]

\[ \bar{V}_v = 3.726 - 0.693 \bar{U}_p - \frac{3.26}{\bar{U}_p} \]
INDUCED POWER CALCULATION

Regime of turbulent wake

\[-1 < \overline{U}_p < 0 \quad y \quad -2 < \overline{V}_v < -1.7 \]

\[\overline{V}_v = -1.7 + 0.3U_p\]

Regime of windmill brake

\[\frac{V_v + v_i}{v_{io}} \cdot \frac{v_i}{v_{io}} = -1\]

\[\begin{align*}
-\overline{V}_v &= \frac{1}{\overline{v}_i} + \overline{v}_i \\
\overline{U}_p &= -\frac{1}{\overline{v}_i} \\
\overline{V}_v &= \overline{U}_p + \frac{1}{\overline{U}_p}
\end{align*}\]
AUTOROTATION
AUTOROTATION

DRIVER DIAGRAM

DRIVEN DIAGRAM

DRIVER ZONE TORQUE = DRIVEN ZONE TORQUE
AUTOROTATION

Stability

\[ dQ_a = dQ_i = r \sin \Phi dL \]
\[ dQ_d = dQ_o = r \cos \Phi dD \]

\[ dQ_a = dQ_d \quad \text{y} \quad \tan |\phi| = \frac{dD}{dL} = \frac{C_L}{C_D} \]