CENTROS DE GRAVEDAD

LONGITUD

$$dm = \lambda\ dl; \quad \lambda : {}^{kg}\!/_{m}; \qquad M = \lambda L$$

$$x_G = \frac{\int x\,dm}{\int dm} = \frac{\lambda \int x\,dl}{\lambda L} = \frac{\int x\,dl}{L}; \quad \int x\,dl = x_G L$$

ÁREA

$$dm = \sigma\ dA; \quad \sigma : {}^{kg}\!/_{m^2}; \qquad M = \sigma A$$

$$x_G = \frac{\int\int\ x\,dm}{\int\int\ dm} = \frac{\sigma \int\int\ x\,dA}{\sigma A} = \frac{\int\int\ x\,dA}{A}; \quad \int\int\ x\,dA = x_G A$$

VOLUMEN

$$dm = \rho\ dv; \quad \rho : {}^{kg}\!/_{m^3}; \qquad M = \rho V$$

$$x_G = \frac{\iiint x\,dm}{\iiint dm} = \frac{\rho \iiint x\,dv}{\rho \iiint\ dv} = \frac{\iiint x\,dv}{V}; \quad \iiint x\,dv = x_G V$$

$$\iiint y\,dv = y_G V \qquad\qquad \iiint z\,dv = z_G V$$

MOMENTOS DE INERCIA

Respecto a un punto: $\iiint (x^2 + y^2 + z^2)dv = I_0$

Respecto a los ejes coordenados:

$$\iiint (y^2 + z^2)dv = I_{0X}; \qquad \iiint (x^2 + z^2)dv = I_{0Y}; \qquad \iiint (x^2 + y^2)dv = I_{0Z}$$

Respecto a los planos coordenados

$$\iiint x^2\ dv = I_{Y0Z} \qquad \iiint y^2\ dv = I_{X0Z} \qquad \iiint z^2\ dv = I_{X0Y}$$

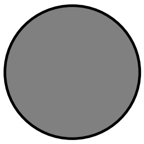
MOMENTOS DE INERCIA DE FIGURAS GEOMÉTRICAS

Varilla de masa M y longitud L

$$I_0 = \frac{ML^2}{3}; \quad I_G = \frac{ML^2}{12}$$

Circulo de radio R

$$I_G = I_{GZ} = \frac{1}{2}MR^2$$



$$I_{GX} = I_{GY} = \frac{1}{4}MR^2$$

Rectángulo de base a y altura b

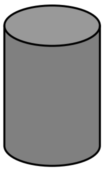
$$I_G = I_{GZ} = \frac{M(a^2+b^2)}{12} \qquad I_{GX} = \frac{Mb^2}{12} \qquad I_{GY} = \frac{Ma^2}{12}$$

$$I_0 = I_{0Z} = \frac{M(a^2+b^2)}{3}; \qquad I_{0X} = \frac{Mb^2}{3} \qquad I_{0Y} = \frac{Ma^2}{3}$$

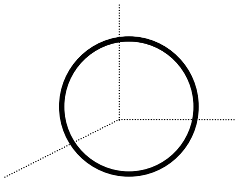
Cilindro de radio R y altura H

$$I_G = I_{XGY} + I_{GZ}$$

$$I_G = \frac{MR^2}{2} + \frac{MH^2}{12}$$



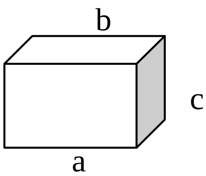
Esfera de radio R



$$I_G = \frac{3MR^2}{5}$$

$$I_{GX} = I_{GY} = I_{GZ} = \frac{2MR^2}{5} \qquad I_{XGY} = I_{XGZ} = I_{YGZ} = \frac{MR^2}{5}$$

Prisma de lados a,b,c



$$I_0 = \frac{M(a^2+b^2+c^2)}{3}$$

$$I_G = I_{XGY} + I_{XGZ} + I_{YGZ} = \frac{M(a^2+b^2+c^2)}{3}$$