Centros de gravedad

Momentos de inercia

Aplicación

Resolución de integrales

Mª Victoria Carbonell

Curso 06/07

Las imágenes de la presentación han sido obtenidas del libro:

Physics for Scientists and Engineers

Paul A. Tipler • Gene Mosca

CENTROS DE GRAVEDAD

LONGITUD

$$dm = \lambda\ dl; \quad \lambda : {}^{kg}\!/_{m}; \qquad M = \lambda L$$

$$x_G = \frac{\int x\,dm}{\int dm} = \frac{\lambda \int x\,dl}{\lambda L} = \frac{\int x\,dl}{L}; \qquad \boxed{\int x\,dl = x_G L}$$

ÁREA

$$dm = \sigma\,dA; \quad \sigma : {}^{kg}\!/_{m^2}; \quad M = \sigma A$$

$$x_G = \frac{\iint x\,dm}{\iint dm} = \frac{\sigma \iint x\,dA}{\sigma \iint dA} = \frac{\iint x\,dA}{A}; \qquad \boxed{\iint x\,dA = x_G A}$$

VOLUMEN

$$dm = \rho\ dv; \quad \rho : {}^{kg}\!/\!{}_{m^3}; \qquad M = \rho V$$

$$x_G = \frac{\iiint x\,dm}{\iiint dm} = \frac{\rho \iiint x\,dv}{\rho \iiint dv} = \frac{\iiint x\,dv}{V}; \qquad \boxed{\iiint x\,dv = x_G V}$$

$$\iiint y\,dv = y_G V$$

$$\iiint z\,dv = z_G V$$

MOMENTOS DE INERCIA

Respecto:
a un punto

$$\iiint (x^2 + y^2 + z^2)dv = I_0$$

a un eje

$$\iiint (x^2 + y^2)dv = I_{0Z}$$

a un plano

$$\iiint x^2 \, dv = I_{Y0Z}$$

MOMENTOS DE INERCIA

Respecto a los ejes

$$\iiint (y^2 + z^2)\,dv = I_{0X}$$

$$\iiint (x^2 + z^2)\,dv = I_{0Y}$$

$$\iiint (x^2 + y^2)\,dv = I_{0Z}$$

MOMENTOS DE INERCIA

Respecto a los planos

$$\iiint x^2 \, dv = I_{Y0Z}$$

$$\iiint y^2 \, dv = I_{X0Z}$$

$$\iiint z^2 \, dv = I_{X0Y}$$

RELACIÓN ENTRE MOMENTOS DE INERCIA

$$I_0 = \frac{1}{2}(I_{0X} + I_{0Y} + I_{0Z})$$

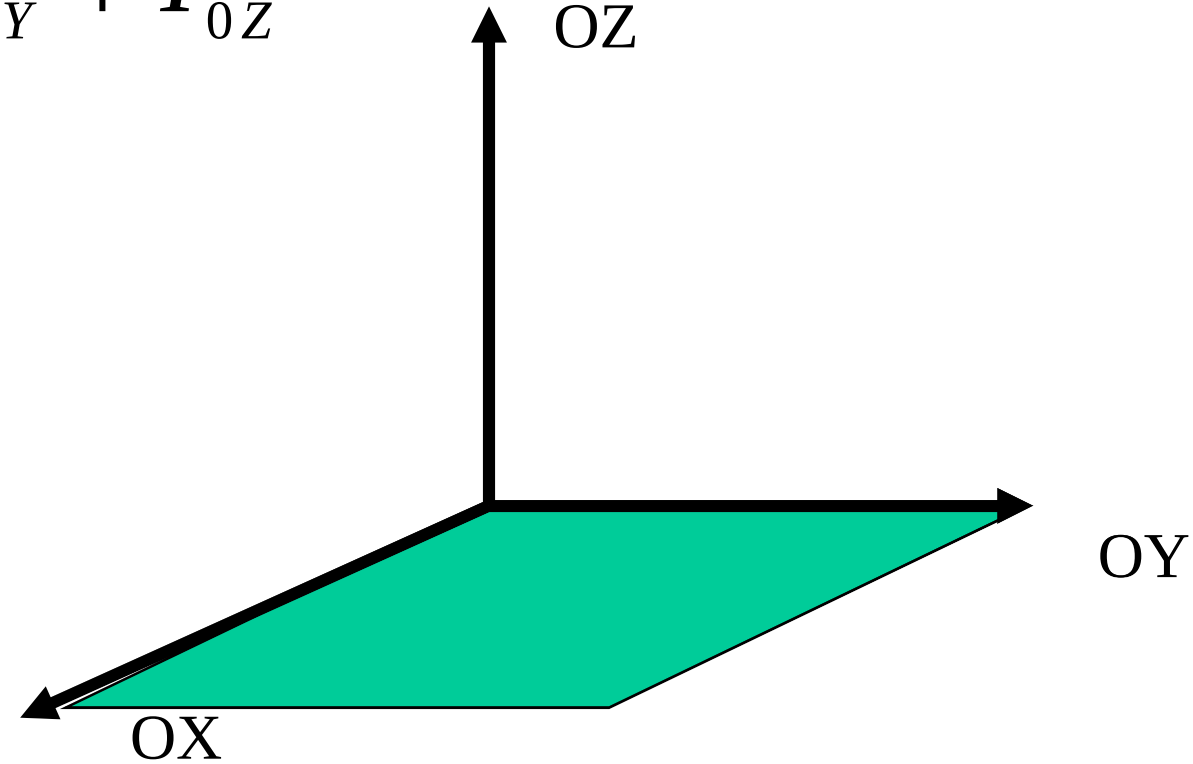
$$I_0 = I_{X0Y} + I_{X0Z} + I_{Y0Z}$$

$$I_0 = I_{X0Y} + I_{0Z}$$

$$I_{0X} = I_{X0Y} + I_{X0Z}$$

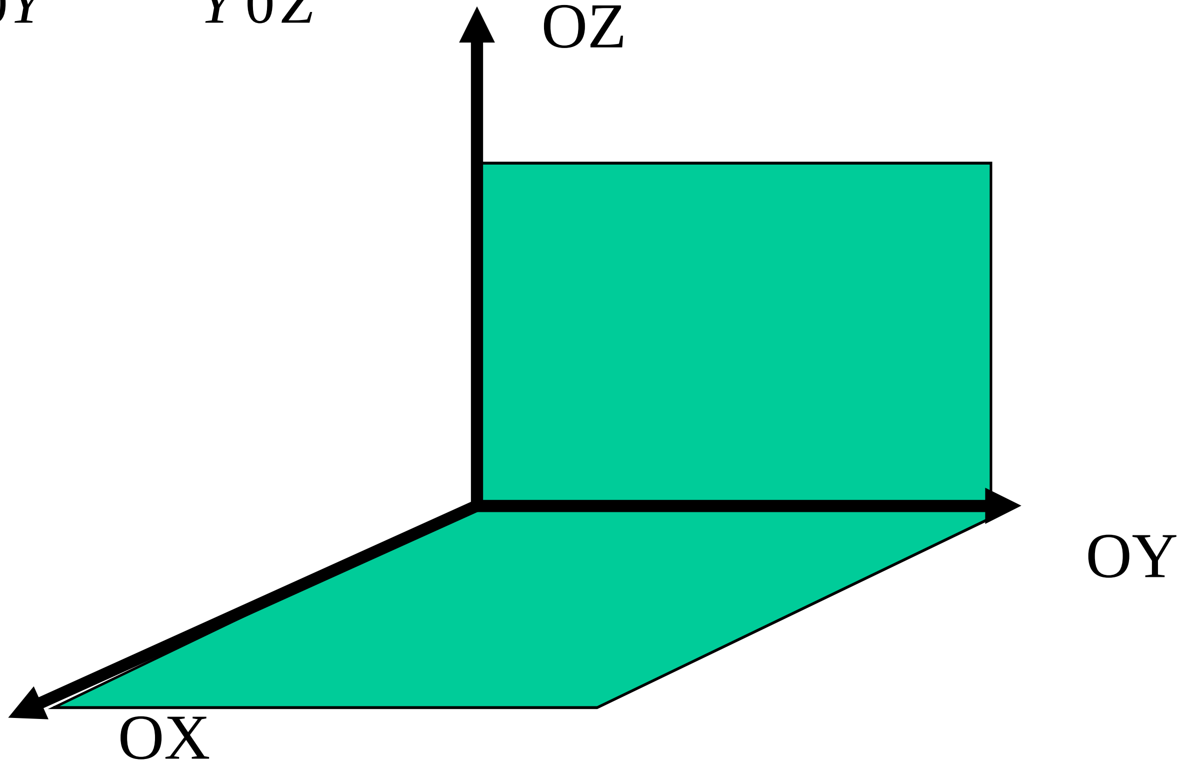
RELACIÓN ENTRE MOMENTOS DE INERCIA

$$I_0 = I_{X0Y} + I_{0Z}$$

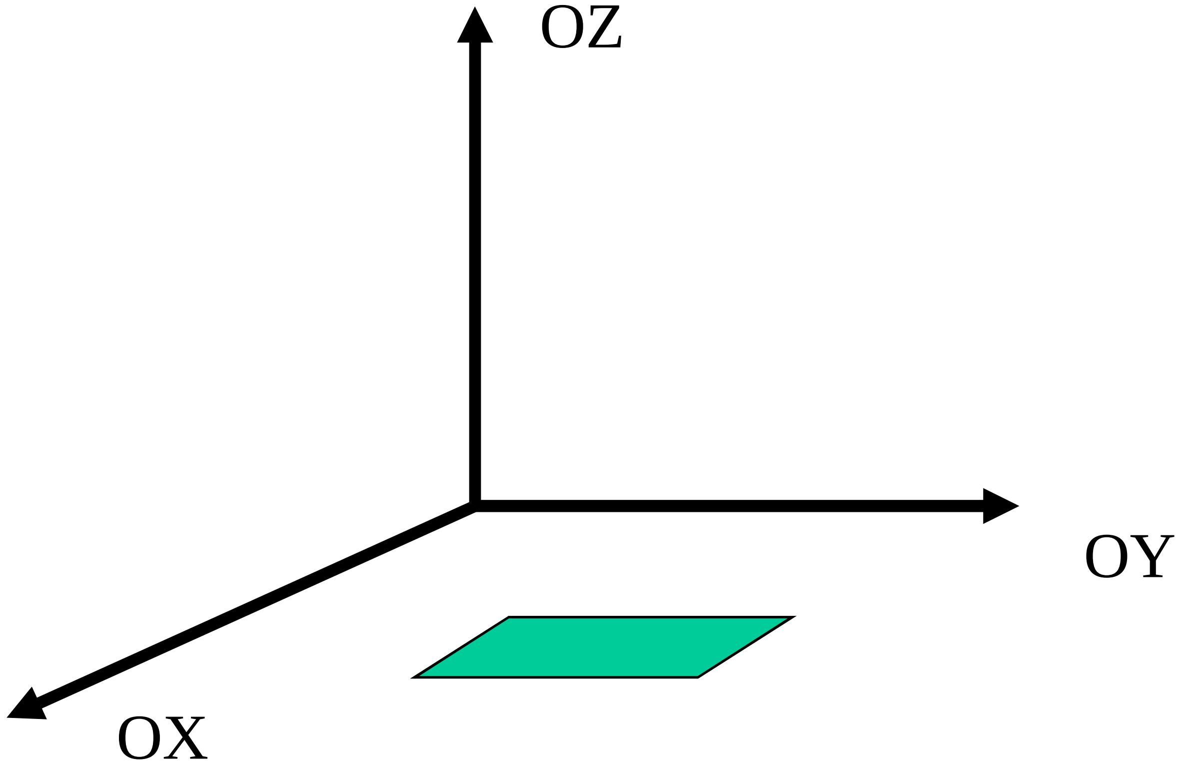


RELACIÓN ENTRE MOMENTOS DE INERCIA

$$I_{0Y} = I_{X0Y} + I_{Y0Z}$$

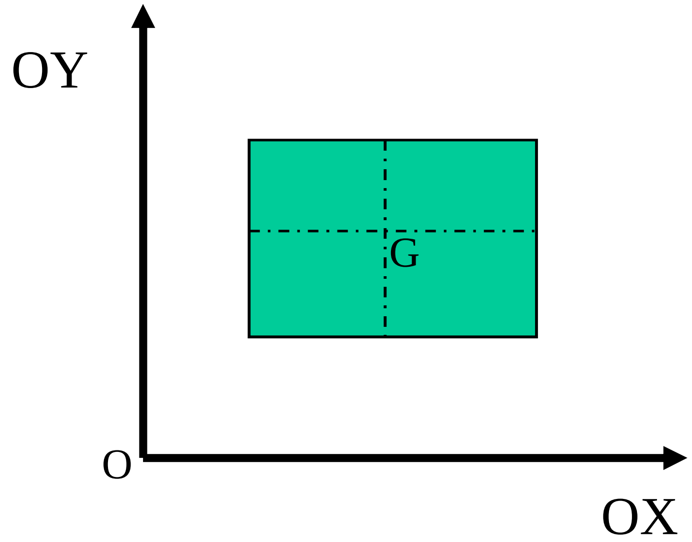


Área plana situada en el plano XOY



Área plana situada en el plano XOY

$$I_0 = I_{X0Y} + I_{0Z}$$



$$I_{X0Y} = 0$$

$$I_0 = I_{0Z}$$

$$I_G = I_{GZ}$$

TEOREMAS DE STEINER respecto a un punto

$$I_0 = I_G + Md^2$$

$$\left\{ \begin{array}{l} I_0 = I_G + Ld^2 \\ I_0 = I_G + \sigma d^2 \\ I_0 = I_G + Vd^2 \end{array} \right.$$

$$d^2 = x^2 + y^2 + z^2$$

TEOREMAS DE STEINER respecto a un eje

$$I_{0X} = I_{GX} + Md^2$$

$$I_{0X} = I_{GX} + Ld^2$$

$$I_{0X} = I_{GX} + \sigma d^2$$

$$I_{0X} = I_{GX} + Vd^2$$

$$d^2 = y^2 + z^2$$

TEOREMAS DE STEINER respecto a un punto

$$I_{X0Y} = I_{XGY} + Md^2$$

$$I_{X0Y} = I_{XGY} + Ld^2$$

$$I_{X0Y} = I_{XGY} + \sigma d^2$$

$$I_{X0Y} = I_{XGY} + Vd^2$$

$$d^2 = z^2$$

MOMENTOS DE INERCIA: longitud

$$dm = \lambda dl; \quad M = \int dm = \lambda L$$

- ***Varilla homogénea de masa M y longitud L***

$$I_0 = \int_0^L x^2 dm = \frac{1}{3} ML^2$$

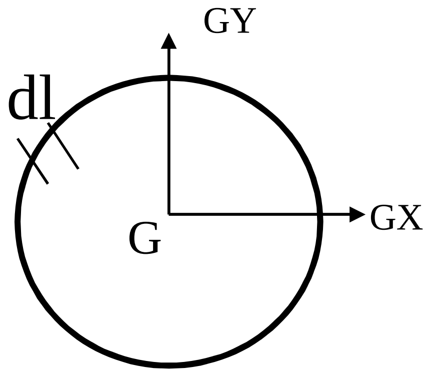
$$I_G = \int_{-L/2}^{L/2} x^2 dm = \frac{1}{12} ML^2$$

$$I_0 = \frac{1}{3} L \cdot L^2$$

$$I_G = \frac{1}{12} L \cdot L^2$$

MOMENTOS DE INERCIA: longitud

- ***Circunferencia homogénea de masa M y longitud L***



$$I_G = \int_0^L r^2 dm = MR^2 = I_{GZ}$$

$$I_{GX} = I_{GY} = \frac{1}{12} MR^2$$

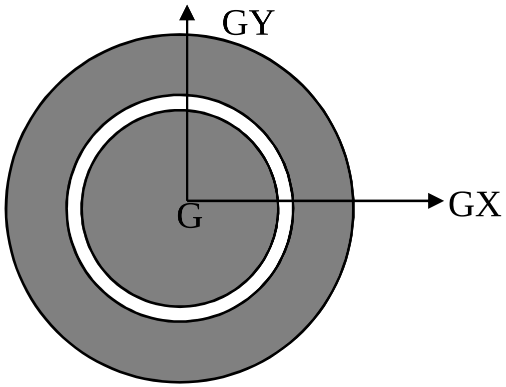
$$I_G = L \cdot R^2 = I_{GZ}$$

$$I_{GX} = I_{GY} = \frac{1}{2} L \cdot R^2$$

MOMENTOS DE INERCIA: área

$$dm = \sigma dA; \quad M = \iint dm = \sigma A$$

- ***Círculo homogéneo de masa M y radio R***



$$dm = \sigma 2\pi r dr$$

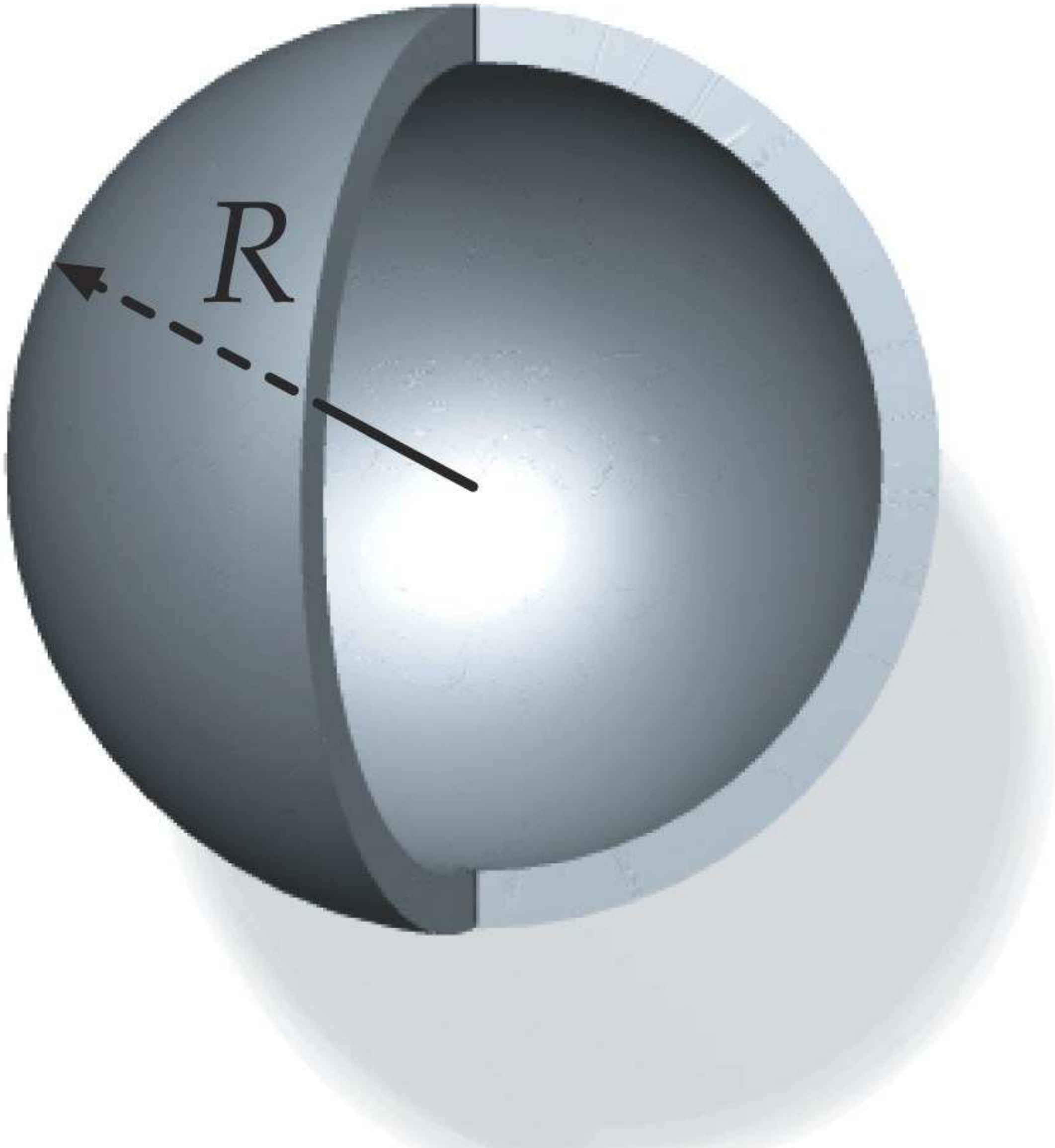
$$I_G = I_{GZ} = \int_0^R r^2 dm = \frac{1}{2} MR^2$$

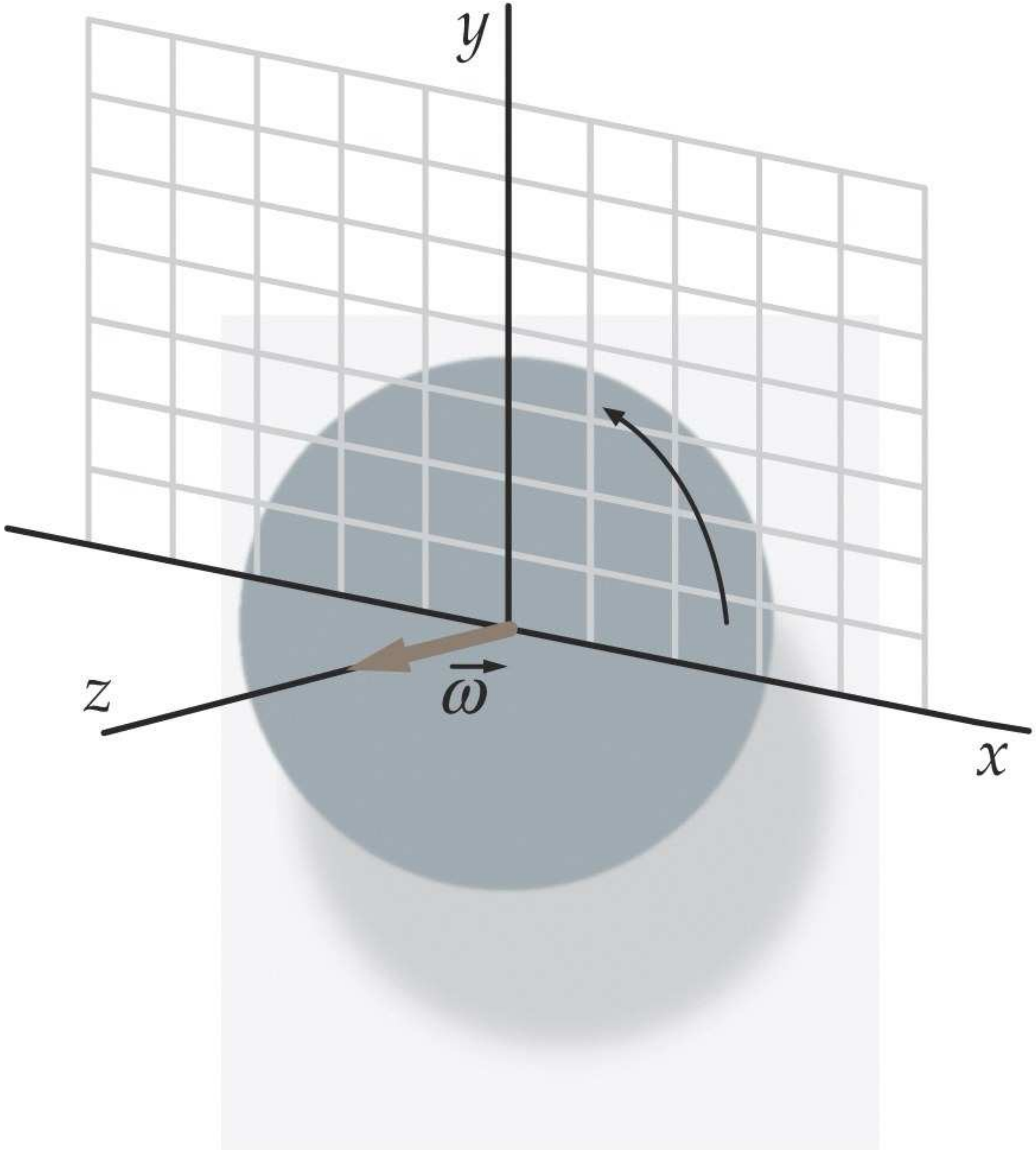
$$I_{GX} = I_{GY} = \int_{-R/2}^{R/2} r^2 dm = \frac{1}{4} MR^2$$

$$I_G = I_{GZ} = \frac{1}{2} A \cdot R^2$$

$$I_{GX} = I_{GY} = \frac{1}{4} A \cdot R^2$$

MOMENTOS DE INERCIA: área (superficie esférica)

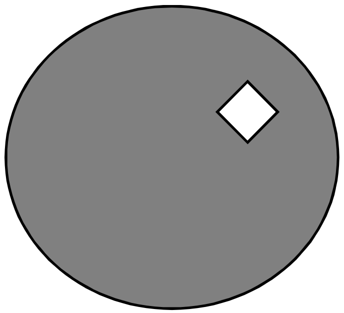




y
z
$\vec{\omega}$
x

MOMENTOS DE INERCIA: área

- *Superficie esférica homogénea de área* **σ** *y radio R*



$$\sigma = 4\pi R^2$$

$$d\sigma = 4\pi r dr$$

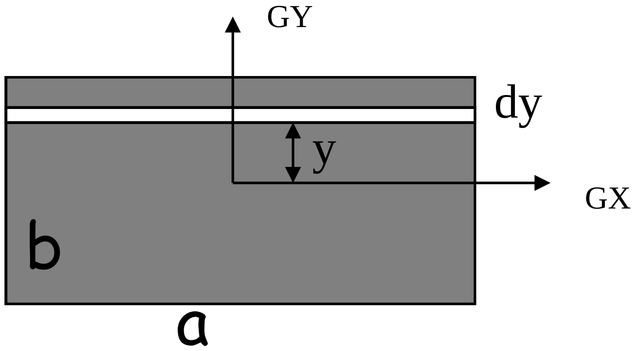
$$I_G = \iint R^2 d\sigma = R^2 \iint d\sigma = \sigma R^2$$

$$I_{GX} = I_{GY} = I_{GZ} = \frac{2}{3}\sigma R^2$$

$$I_{XGY} = I_{XGZ} = I_{YGZ} = \frac{1}{3}\sigma R^2$$

MOMENTOS DE INERCIA: área

- ***Rectángulo homogéneo de lados a, b y área σ***



$$d\sigma = a\,dy$$

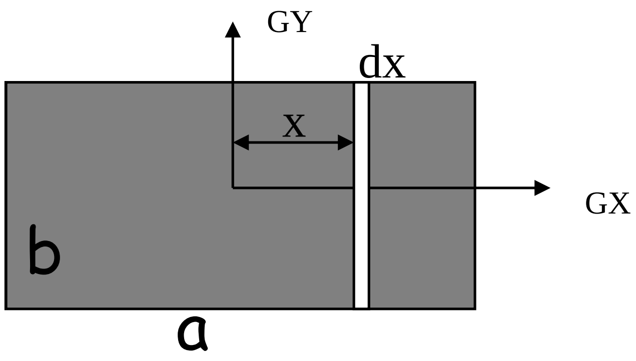
$$\sigma = ab$$

$$I_{GX} = \iint y^2 d\sigma = a\int_{-b/2}^{b/2} y^2 dy = \frac{1}{12}ab^3$$

$$I_{GX} = \frac{1}{12}\sigma b^2$$

MOMENTOS DE INERCIA: área

- *Rectángulo homogéneo de lados a, b y área* $\boldsymbol{\sigma}$

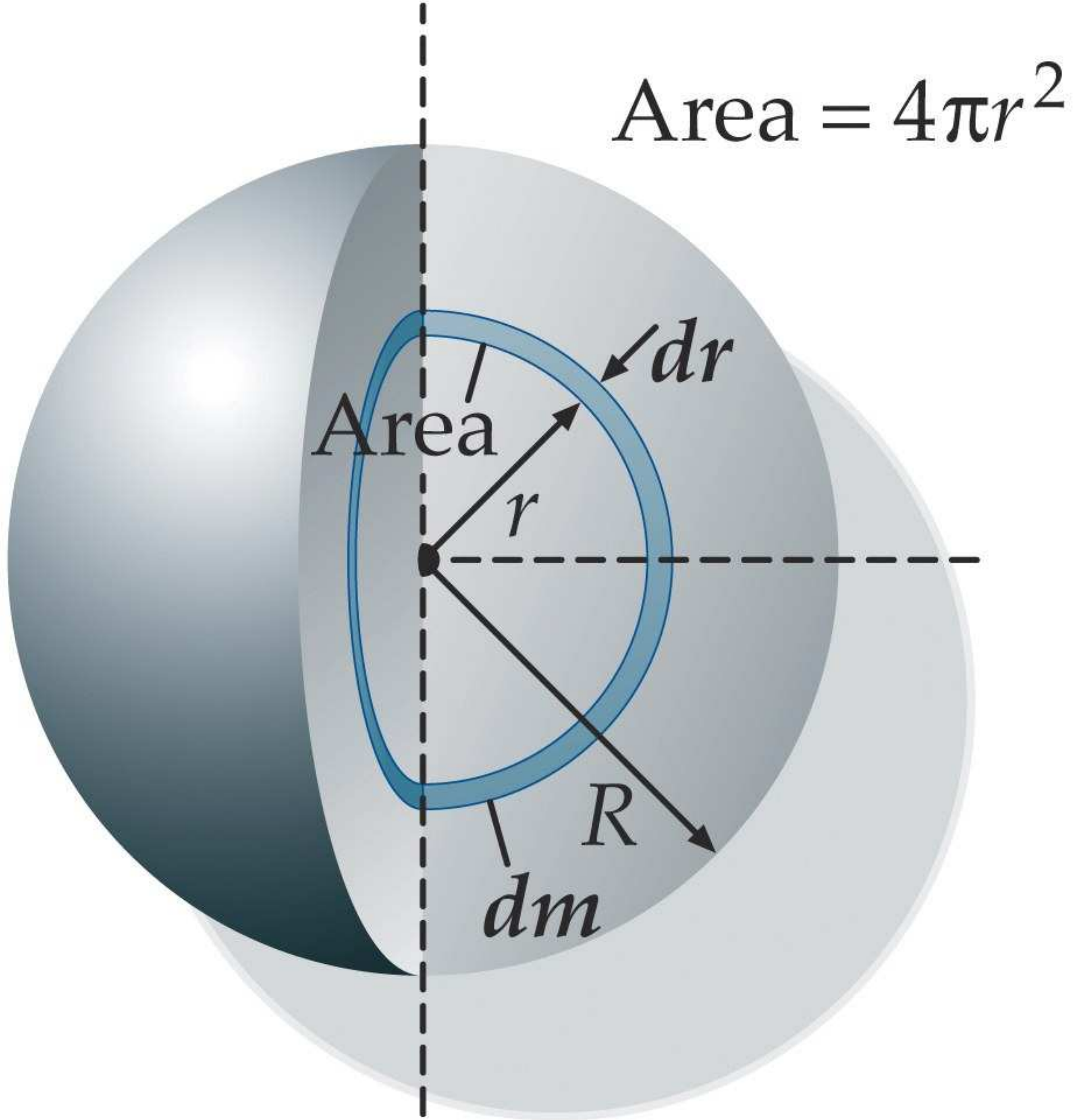


$$d\sigma = b\,dx$$

$$I_{GY} = \iint x^2 d\sigma = b\int_{-b/2}^{b/2} x^2 dx = \frac{1}{12} b a^3$$

$$I_{GY} = \frac{1}{12}\sigma a^2$$

$$I_G = I_{GZ} = \frac{1}{12}\sigma(a^2 + b^2)$$



Area = $4\pi r^2$
Area
dr
r
R
dm

MOMENTOS DE INERCIA: volumen

- *Esfera homogénea de radio R y volumen V*

$$dV = 4\pi r^2 dr$$

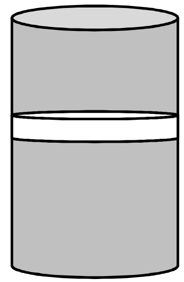
$$I_G = \iiint r^2 dv = 4\pi \int_0^R r^4 dr = \frac{3}{5} VR^2$$

$$I_{GX} = I_{GY} = I_{GZ} = \frac{2}{5} VR^2$$

$$I_{XGY} = I_{XGZ} = I_{YGZ} = \frac{1}{5} VR^2$$

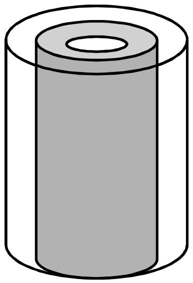
MOMENTOS DE INERCIA: volumen

- *Cilindro de radio R y altura H*



$$dv = \pi r^2 dz$$

$$I_{XGY} = \iiint z^2 dv = \pi R^2 \int_{-H/2}^{H/2} z^2 dz = \frac{1}{12} VH^2$$



$$dv = 2\pi r H dr$$

$$I_{GZ} = \iiint r^2 dv = 2\pi H \int_0^R r^3 dr = \frac{1}{2} VR^2$$

$$I_G = \frac{1}{2} VR^2 + \frac{1}{12} VH^2$$