Radiation Pressure

Modeling the Space Environment

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April 2008
Radiation Pressure Propels Solar Sails
Radiation Pressure

Electromagnetic radiation produces a force over the satellite

<table>
<thead>
<tr>
<th>Sun</th>
<th>Earth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar radiation:</td>
<td>Albedo: reflection and scattering of incident solar radiation</td>
</tr>
<tr>
<td>electromagnetic radiation from X-ray to radiofrequency</td>
<td>(\sim 10 - 35% R_s)</td>
</tr>
<tr>
<td>Solar wind:</td>
<td>Earth’s IR re-emission</td>
</tr>
<tr>
<td>charged particles (mainly protons) and electrons</td>
<td></td>
</tr>
</tbody>
</table>
Radiation Pressure Cause

- Electromagnetic radiation carries:
  - Energy
  - Linear Momentum

- Each photon has:
  - Speed
  - Energy
  - Linear momentum

- Solar flux $\Phi$:
  - Energy flux per unit normal area at 1 AU from the Sun

\[
\Phi = \frac{\Delta E}{A \Delta t} \approx 1367 \text{ W/m}^2
\]

Light speed: $c = 2,9979250 \cdot 10^8 \text{ m/s}$
Photon’s frequency: $\nu$

Plank’s constant: $\hbar = 6.626068 \cdot 10^{-34} \text{ m}^2\text{kg/s}$
Impact Types

Specular reflection: \[ \mathbf{p}_2 = \mathbf{p}_1 + 2\mathbf{p}_1 \cos \theta \mathbf{n} \]
Reflectivity: \[ \epsilon = 1 \]
Rad. Pressure Coefficient: \[ C_R = 1 + \epsilon = 2 \]

Diffuse reflection: \[ \overline{\mathbf{p}}_2 = \frac{\mathbf{p}_1}{2} \mathbf{n} \]
Reflectivity: \[ \epsilon = 0 - 1 \]
Rad. Pressure Coefficient: \[ C_R = 1 - 2 \]

Absorption: \[ \mathbf{p}_2 = 0 \]
Reflectivity: \[ \epsilon = 0 \]
Rad. Pressure Coefficient: \[ C_R = 1 \]

Transparency: \[ \mathbf{p}_2 = \mathbf{p}_1 \]
Reflectivity: \[ \epsilon = -1 \]
Rad. Pressure Coefficient: \[ C_R = 0 \]
Theoretical Computation of Radiation Pressure

- Force over an element of surface:

\[ d\mathbf{F}_{inc} = (p_1 - p_2) N \cdot ds = \frac{(p_1 - p_2)}{p_1} \max\left(-\Phi \cdot ds, 0\right) \]

\[ N \quad \text{Number or particles per unit time and unit normal area} \]

\[ \Phi \quad \text{Solar energy flux} \]

\[ \Phi/c \quad \text{Linear momentum flux:} \quad \frac{\Phi}{c} \simeq 4.56 \cdot 10^{-6} \text{ N/m}^2 \]

- Integrate over all the surface exposed to radiation
- Include radiation reflected by other parts of the satellite
- Thermal emission: \[ d\mathbf{F}_{emi} = -a\sigma T^4 ds/c \]
  \[ (\sigma = 5.661 \cdot 10^{-8} \text{ WK}^4/\text{m}^2, \text{Boltzmann constant}) \]

- Radioelectric emission: \[ F_{emi} \simeq \dot{W}/c \]
Simple Model of Radiation Pressure

- For simple numerical simulation: global coefficient $C_R$:

$$ F_{rad} = \nu P_{rad} C_R A_{\odot} $$

- Assume force along flux vector $-r_{\odot}$
- RP Coeff $C_R \simeq 1 - 2$ (Differential determination)
- Radiation pressure: $P_{rad} = \frac{\Phi}{c} = \frac{\Phi}{c} \frac{-r_{\odot}}{r_{\odot}}$

$$ (P_{rad} = 4.56 \cdot 10^{-6} \text{ N/m}^2) $$

- Area exposed to sun $A_{\odot}$ ($\neq$ ram area for drag)
- Area exposed changes with attitude/Solar panels do not
- Shadow function $\nu$: Earth shadow cone blocks the Sun
- Radiation pressure torque $\rightarrow$ rotation/attitude control
Detailed Model of Radiation Pressure

- Account for the reflection/absorption and surface orientation of each section of the satellite

\[ \mathbf{F}_i = -\nu P_{rad} \cos \theta_i A_i \left[ (1 - \epsilon_i) \mathbf{e}_\odot + 2\epsilon_i \cos \theta_i \mathbf{n}_i \right] \]

- Shadow function \( \nu \) with light/umbra/penumbra
- Must know satellite attitude, shape \( \mathbf{n}_i, \cos \theta_i \) and surfaces \( \epsilon_i \)
- Reflections from other parts of the satellite
- Requires a detailed model of the satellite (FEA)

<table>
<thead>
<tr>
<th>Material</th>
<th>( \epsilon )</th>
<th>( 1 - \epsilon )</th>
<th>( C_R \simeq 1 + \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar panel</td>
<td>0.21</td>
<td>0.79</td>
<td>1.21</td>
</tr>
<tr>
<td>H-G antenna</td>
<td>0.30</td>
<td>0.70</td>
<td>1.30</td>
</tr>
<tr>
<td>Al-Mylar solar sail</td>
<td>0.88</td>
<td>0.12</td>
<td>1.88</td>
</tr>
</tbody>
</table>
Shadow Function: Cylindrical

Simplified shadow function: cylindrical shadow cone:

\[ \nu = \begin{cases} 
1 & \text{Lighted} \\
0 & \text{Inside shadow cylinder}
\end{cases} \]

Cf. Vallado
Shadow Function: Umbra/Penumbra

Regions of the shadow cone: lighted, umbra, penumbra

\[ \nu = \begin{cases} 
1 & \text{Lighted} \\
\in [0, 1] & \text{Penumbra, outer umbra (217 ER, too far!)} \\
0 & \text{Umbra} 
\end{cases} \]

Penumbra: compute visible fraction of Sun surface

Cf. Montenbruck
Solar Flux

- **Constant model:**
  - Mean solar flux at 1 AU (ESA):
  - Max solar flux (summer solstice):
  - Min solar flux (winter solstice):

- **Annual variation model:**
  
  \[
  \Phi = \frac{1358}{1.004 + 0.0334 \cos D_{aph}} \quad \text{W/m}^2
  \]

  \(D_{aph}\): annual phase. Angle from Aphelium (approx. 4th July)
Time changes of Flux:
- Summer-Winter: 3.4%
- 11-year Sun cycle: 0.1%
- UV+ part very variable ($F_{10.7}$), but holds little energy

<table>
<thead>
<tr>
<th>Type</th>
<th>Wavelength (nm)</th>
<th>Average $\Phi$ (W/m$^2$)</th>
<th>Worst-case $\Phi$ (W/m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near UV</td>
<td>180-400</td>
<td>118</td>
<td>177</td>
</tr>
<tr>
<td>UV</td>
<td>&lt; 180</td>
<td>$2.3 \cdot 10^{-2}$</td>
<td>$4.6 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>UV</td>
<td>100-150</td>
<td>$7.5 \cdot 10^{-3}$</td>
<td>$1.5 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>EUV</td>
<td>10-100</td>
<td>$2 \cdot 10^{-3}$</td>
<td>$4 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>X-rays</td>
<td>1-10</td>
<td>$5 \cdot 10^{-5}$</td>
<td>$1 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>Flare X-rays</td>
<td>0.1-1</td>
<td>$1 \cdot 10^{-4}$</td>
<td>$1 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>
Solar radiation pressure: (pure absorption: $C_R = 1$)

$$P_{rad} = \frac{\Phi}{c} = 4.51 \cdot 10^{-6} \frac{\text{W s}}{\text{m}^3} \left( \frac{\text{N}}{\text{m}^2} \right)$$

Earth Radiation Pressure

- Earth albedo (only day side): $< 475 \text{ W/m}^2$, average: $0.3\Phi$
- IR emission (10-20% day/night): $< 260 \text{ W/m}^2$, average: 230
- Changes with Earth surface: Sea, ice, land... Must divide Earth in surface elements...
- Spherical Harmonics model for emissivity
- Only for very high precision: diminishes with height

Divide Earth in $N$ surface elements $i$ (about 20)

$$\vec{r}_{ERP} = \sum_{i=1}^{N} C_{R} \left( \nu_{i} a_{i} \cos \theta_{i}^{\oplus} + \frac{1}{4} \epsilon_{i} \right) P_{rad} \frac{A_{S}}{m} \cos \theta_{i}^{S} \frac{dA_{i}^{\oplus}}{\pi r_{i}^{2}} \, e_{i}$$

- $\nu_{i}$: shadow function of surface $i$: \begin{align*}
\text{Albedo:} & \quad \text{See Sun and Sat} \\
\text{IR:} & \quad \text{See Satellite}
\end{align*}
- $a_{i}$: albedo/IR factor: average 0.34, changes over the Earth
- $\epsilon_{i}$: emissivity of surface element, average 0.68-disk/sphere
- $\cos \theta_{i}^{\oplus}$: angle of Earth surface element $i$ with Sun
- $\cos \theta_{i}^{S}$: angle of satellite surface with Earth element $i$
- $e_{i}$: unit vector from surface element to satellite at a distance $r_{i}^{2}$
- Compute separately for Albedo and IR
Albedo variability with latitude and season:

\[ a = a_0 + a_1 P_1 (\sin \phi) + a_2 P_2 (\sin \phi) \]

\[ a_1 = c_0 + c_1 \cos [\omega (JD - t_0)] + c_2 \sin [\omega (JD - t_0)] \]

where

- \( t_0 \) Epoch
- \( \omega \) Earth orbit pulsation, \( 2\pi/365.25 \)
- \( \phi \) Equatorial geocentric latitude
- JD Julian Date
- \( P_n \) Legendre polynomial of degree \( n \)
- \( a_0 = 0.34 \), \( a_2 = 0.29 \)
- \( a_1 : c_0 = 0 \), \( c_1 = 0.1 \), \( c_2 = 0 \)

Longitude considered through Sun angle and shadow function.
Earth IR radiation variability with latitude and season:

\[ e = e_0 + e_1 P_1 (\sin \phi) + e_2 P_2 (\sin \phi) \]

\[ e_1 = k_0 + k_1 \cos [\omega (JD - t_o)] + k_2 \sin [\omega (JD - t_o)] \]

where

- \( t_o \) Epoch
- \( \omega \) Earth orbit pulsation, \( 2\pi / 365.25 \)
- \( \phi \) Equatorial geocentric latitude
- \( JD \) Julian Date
- \( P_n \) Legendre polynomial of degree \( n \)
- \( e_0 = 0.68 \), \( e_2 = -0.18 \)
- \( e_1 : k_0 = 0 \), \( k_1 = -0.07 \), \( k_2 = 0 \)
Effects of Radiation Pressure

- Small, except for light satellites; important in GEO
- Periodic changes in all elements (yearly: $e$, orbital: $a$)
- Secular changes in $\Omega$, $\omega$
- Small change with solar activity
- Comparison with atmospheric drag:

$$\frac{a_{aerod}}{a_{rp}} = \frac{1}{2} \rho \frac{C_D A}{m} \frac{v_{rel}^2}{P_{rad} C_R A_\odot/m} \approx \frac{\rho v_{rel}^2}{P_{rad}^2} \Rightarrow \text{equal at } \approx 800\text{km}$$

$A$ and $A_\odot$ assumed to be similar.

- Application:
  - Propulsion: solar sails
  - Maneuver: flaps in solar panels
Visualization of Radiation Pressure Effects

[Diagram showing the visualization of radiation pressure effects with arrows indicating the direction of pressure.]
Visualization of Radiation Pressure Effects

$-\Delta \nu$
Visualization of Radiation Pressure Effects

\[-\Delta \nu\]
Visualization of Radiation Pressure Effects
Periodic variation of vector $\mathbf{e}$ normal to the sun vector. After one year, it returns to original value.
# VOP Effects of Perturbations

<table>
<thead>
<tr>
<th></th>
<th>Gravity</th>
<th>3rd Body</th>
<th>Atm Drag</th>
<th>Rad Press</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Zonal</td>
<td>Sect/Tess</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a$</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>S</td>
</tr>
<tr>
<td>$e$</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>S</td>
</tr>
<tr>
<td>$i$</td>
<td>P</td>
<td>P</td>
<td>P</td>
<td>S</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>P</td>
<td>S</td>
<td>P</td>
<td>S</td>
</tr>
<tr>
<td>$\omega$</td>
<td>P</td>
<td>S</td>
<td>P</td>
<td>S</td>
</tr>
<tr>
<td>$M_0$</td>
<td>P</td>
<td>S</td>
<td>P</td>
<td>S</td>
</tr>
</tbody>
</table>

P: Periodic  
S: Secular  
Also: coupling effects

Source: Vallado